Prediction of Wave-Induced Motions for Hullborne Hydrofoils

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Linear mathematical models are presented for predicting sway, heave, roll, pitch, and yaw motions of hullborne hydrofoil ships at arbitrary headings to the sea. These models are derived by combining linearized hydrofoil terms with standard strip theory. Predictions generally agree well with experiments.

Nomenclature

A_{jk} = added mass coefficient
B_{jk} = damping coefficient
C_{jk} = restoring coefficient
C_D = drag coefficient
C_L = lift coefficient
$C_{L\alpha}$ = lift curve slope
C_n = flat-plate normal force coefficient
C_w = strut wave-making damping coefficient
$C(k)$ = Theodorsen's function
F = superscript denoting foil contribution
F_j = exciting force or moment
$G(k)$ = gust function
H = superscript denoting hull contribution
I_4 = rolling moment of inertia
I_5 = pitching moment of inertia
I_6 = yawing moment of inertia
L = foil lift; also length between perpendiculars
M_{jk} = generalized mass
N = foil yawing moment
S = foil area
U = ship speed
b = foil span
c = foil mean chord
g = gravitational acceleration
h = foil mean depth
k = reduced frequency
k_w = wave number
m = ship mass
s = x coordinate of foil midchord
t = time variable
u = wave horizontal orbital velocity
w = wave vertical orbital velocity
$\hat{w}$ = component of wave orbital velocity perpendicular to the foil
x, y, z = coordinate system
$\hat{z}$ = heave amplitude
Γ = foil dihedral angle
α = foil angle of attack
β_s = seaway direction
ζ = wave amplitude
η = wave elevation
η_2 = sway displacement
η_3 = heave displacement
η_4 = roll angle
$\hat{\eta}_4$ = roll amplitude
η_5 = pitch angle
η_6 = yaw angle
$\hat{\theta}$ = pitch amplitude
ρ = density of water
ω = frequency of encounter
ω_w = wave frequency

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I. Introduction

As open ocean military hydrofoil ships may spend a significant portion of their operational time in the hullborne mode, there is a clear need for reliable theoretical methods of predicting wave-induced motions for hullborne hydrofoils. This paper addresses the foregoing requirement and presents linear mathematical models for predicting sway, heave, roll, pitch, and yaw motions at arbitrary headings to the sea. The basic theoretical tools are drawn from the fields of displacement ship seakeeping and hydrofoil dynamic simulation, where they have been well established individually. Hull exciting forces, added mass, and damping are computed by the usual means of strip theory. Upon these are superposed linearized hydrofoil terms, derived by incorporating the effects of unsteady hydrodynamics into a quasisteady formulation. Hull-foil interaction is neglected, and viscous effects are included only at zero speed.

Predictions are compared with experimental results for three vastly different hydrofoil ships. One is the Canadian hydrofoil ship *Bras d'Or*, with a surface-piercing foil system, for which full-scale trials data are available. The other two are the American hydrofoils *PHM* and *AG(EH)*, which have fully submerged foil systems, one canard and the other airplane, and greatly different hulls; for these, model tests are the source of data.

Predictions generally agree well with experiment. The good agreement between predicted and measured vertical motions suggests that pitch and heave predictions for hullborne hydrofoils should be of comparable accuracy to strip theory predictions for conventional displacement hulls. As to lateral motions, although predictions agree well with the measurements available, the experimental data are not sufficiently extensive to permit meaningful assessment of the general reliability of predictions.

II. Theoretical Model

The most important assumptions and restrictions made in deriving the theoretical model are listed below:

1) Ship response is a linear function of wave excitation.
2) Ship length is much greater than either beam or draft.
3) Surging effects are negligible; forward velocity is either zero or much larger than other ship velocity components and wave orbital velocities.
4) The hull does not develop appreciable planing lift.
5) All viscous effects are negligible except for zero speed foil and strut damping, where quadratic damping is assumed.
6) Hull-foil interaction is negligible.

In applying strip theory to a displacement hull, assumptions 1-4 normally are assumed, but assumption 5 is changed to "all viscous effects other than roll damping are negligible," and the affect of viscosity on roll damping is included at all speeds. For hydrofoil ships, however, which do not have bilge keels, hull viscous damping is always negligible compared

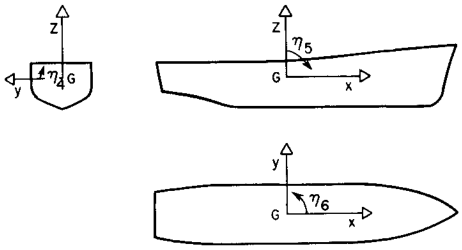

Fig. 1 Axis system.

with foil and strut damping, and viscosity has a significant effect on the latter only in the neighborhood of zero speed. Assumption 6 makes the problem theoretically tractable by permitting direct superposition of hull and foil terms.

The axis system is illustrated in Fig. 1. (x, y, z) is a right-handed orthogonal coordinate system fixed with respect to the mean position of the ship with the origin at the mean position of the center of gravity. The positive x axis points forward in the direction of motion, the positive y axis to port, and the positive z axis vertically upward. Sway is denoted by η_2, heave by η_3, roll by η_4, pitch by η_5, and yaw by η_6. The ship is assumed to be traveling at constant speed U along a mean course at angle β_s to the direction of propagation of a train of long-crested waves (Fig. 2). Note that for head seas $\beta_s = 180°$ and for beam seas $\beta_s = 90°$.

Under the assumptions that the oscillatory motions are linear and harmonic, and that the ship has lateral symmetry, two sets of coupled equations are obtained: one set describes heave and pitch motions; the other describes sway, roll, and yaw. Each equation has the following general structure

$$\sum_k (M_{jk}+A_{jk})\ddot{\eta}_k + (B_{jk}\dot{\eta}_k + C_{jk}\eta_k) = F_j e^{i\omega t} \tag{1}$$

M_{jk} are the components of the generalized mass matrix for the ship; A_{jk}, B_{jk}, and C_{jk} are the added mass, damping, and restoring coefficients; F_j are the complex amplitudes of the exciting force and moment. Each of the A_{jk}, B_{jk}, C_{jk}, and F_j are ascribed the following general form

$$F_j = F_j^H + F_j^F \tag{2}$$

where F_j^H and F_j^F denote contributions from the hull and foils, respectively. Expressions for the F_j^H, F_j^F, etc., are given below.

Longitudinal Motions

The coupled heave and pitch equations are

$$(A_{33}+m)\ddot{\eta}_3 + B_{33}\dot{\eta}_3 + C_{33}\eta_3 + A_{35}\ddot{\eta}_5 + B_{35}\dot{\eta}_5 + C_{35}\eta_5 = F_3 e^{i\omega t} \tag{3}$$

$$A_{53}\ddot{\eta}_3 + B_{53}\dot{\eta}_3 + C_{53}\eta_3 + (A_{55}+I_5)\ddot{\eta}_5 + B_{55}\dot{\eta}_5 + C_{55}\eta_5 = F_5 e^{i\omega t} \tag{4}$$

where m is ship mass and I_5 the pitching moment of inertia. The subscript conventionis the same as in Salvesen et al.[1]

Hull coefficients

Hull coefficients are evaluated by the usual means of strip theory. Specifically, the formulation of Salvesen et al.[1] is used, and mathematical expressions for the hull coefficients are obtained therefrom; these will not be reproduced here.

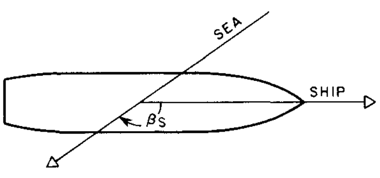

Fig. 2 Definition of sea direction.

Foil coefficients

For the case of nonzero forward speed, the foil coefficients are derived by incorporating finite span and free-surface correction factors into the general structure of Theodorsen's classical solution of the two-dimensional problem of a pitching and heaving aerofoil.[2] This procedure now will be described.

Denote by L_d and M_d the lift and pitching moment acting on a hydrofoil as a result of pitching and heaving motions. Then, from Bisplinghoff et al.[2] with simple corrections for finite depth and aspect ratio

$$L_d = L_{NC} + L_C \tag{5}$$

$$M_d = -L_{NC}s - L_C x - \pi\rho b(c^3/16)[U\dot{\eta}_5 + (c/8)\ddot{\eta}_5] \tag{6}$$

where the subscript NC denotes noncirculatory and C circulatory. L_{NC} is an added mass term, whereas L_C consists of dynamic angle-of-attack terms modified to account for circulation delay

$$L_{NC} = \pi\rho b(c/2)^2(s\ddot{\eta}_5 - U\dot{\eta}_5 - \ddot{\eta}_3) \tag{7}$$

$$L_C = \tfrac{1}{2}\rho USC_{L\alpha}C(k)[(s-c/4)\dot{\eta}_5 - U\eta_5 - \dot{\eta}_3] - (\partial L/\partial h)C(k)(\eta_3 - x\eta_5) \tag{8}$$

where c is mean chord, $C_{L\alpha}$ lift curve slope, b and S horizontally projected span and area, respectively, and x and s the x coordinates of quarter- and midchord, respectively. $C_{L\alpha}$ is evaluated, using the procedure of Jones et al.,[3] as a function of aspect ratio, immersion depth, section geometry, and forward speed. $\partial L/\partial h$ then is evaluated by numerical differentiation of $C_{L\alpha}$

$$\frac{\partial L}{\partial h} = \frac{1}{2}\rho U^2 S\left[\frac{\partial C_{L\alpha}}{\partial h}\right]_0 \alpha_0 \tag{9}$$

where subscript 0 denotes the reference condition of straight and level flight in calm water;

$$k = \omega c/(2U) \tag{10}$$

is the reduced frequency, and $C(k)$ accounts for circulation delay. Evaluation of $C(k)$ is based on the classical work of Jones,[4] with modifications developed by Drummond et al.[5] Note that the unsteady lift function is three-dimensional; Theodorsen's two-dimensional formulation only provides the basis for the general mathematical structure of the problem.

The following foil coefficients are obtained from Eqs. (5-8):

$$A_{33}^F = \pi\rho\Sigma b(c/2)^2 \tag{11}$$

$$B_{33}^F = \tfrac{1}{2}\rho U\Sigma SC_{L\alpha}C(k) \tag{12}$$

$$C_{33}^F = \Sigma\frac{\partial L}{\partial h}C(k) \tag{13}$$

$$A_{35}^F = A_{53}^F = \pi\rho\Sigma b(c/2)^2 s \tag{14}$$

$$B_{53}^F = -\tfrac{1}{2}\rho U\Sigma SC_{L\alpha}C(k)x \tag{15}$$

$$B_{35}^F = UA_{33}^F \mp B_{53}^F \tag{16}$$

$$C_{53}^F = -\Sigma\frac{\partial L}{\partial h}C(k)x \tag{17}$$

$$C_{35}^F = UB_{33}^F + C_{53}^F \tag{18}$$

$$A_{55}^F = \pi\rho\Sigma b\left(\frac{cs}{2}\right)^2 + \frac{\pi\rho}{128}\Sigma bc^4 \tag{19}$$

$$B_{55}^F = UA_{35}^F + \frac{1}{2}\rho U\Sigma SC_{L\alpha}C(k)x\left(s-\frac{c}{4}\right) + \frac{\pi\rho U}{16}\Sigma bc^3 \tag{20}$$

$$C^F_{55} = UB^F_{53} + \Sigma \frac{\partial L}{\partial h} C(k)x^2 \quad (21)$$

where summation is over all foil elements.

Consider now the foil exciting force and moment, and denote by L_w and M_w the lift and pitching moment due to wave action on the foil. Then

$$L_w = \tfrac{1}{2}\rho USC_{L\alpha}G(k)w + (\partial L/\partial h)C(k)\eta \quad (22)$$

$$M_w = -xL_w \quad (23)$$

where $G(k)$ is Jones' gust function as modified by Drummond et al., and η and w are the wave elevation and vertical component of wave orbital velocity at quarter-chord. w is related to η by

$$w = i\omega_w \exp(-k_w h)\eta \quad (24)$$

where ω_w is wave frequency, related to frequency of encounter by

$$w = \omega_w - k_w U \cos\beta_s \quad (25)$$

and where

$$k_w = \omega_w^2/g \quad (26)$$

is wave number and h is foil mean depth.

From Eqs. (22) to (24), the foil exciting force and moment are

$$f^F_3 = [\tfrac{1}{2}\rho USC_{L\alpha}G(k)i\omega_w \exp(-k_w h) + (\partial L/\partial h)C(k)]$$
$$\exp(-ik_w x\cos\beta_s) \quad (27)$$

$$F^F_3 = \Sigma f^F_3 \quad (28)$$

$$F^F_5 = -\Sigma f^F_3 x \quad (29)$$

The case of zero forward speed merits special consideration. Viscous drag forces opposing relative motion then act on the foils. For a given foil, the viscous force may be expressed as

$$F_v = \tfrac{1}{2}\rho SC_D(w - \dot\eta_3 + s\dot\eta_5)^2$$
$$= \tfrac{1}{2}\rho SC_D(w^2 + \dot\eta_3^2 + s^2\dot\eta_5^2 - 2w\dot\eta_3 + 2sw\dot\eta_5 - 2s\dot\eta_3\dot\eta_5) \quad (30)$$

where C_D is the drag coefficient. Examination of Hoerner[6] suggests a value of 1.17 for C_D.

The terms involving w represent viscous contributions to the exciting forces. It is standard practice in linear ship motion theory to ignore these and to include the effects of viscosity only on the major damping terms, in this case B_{33} and B_{55}. In keeping with this philosophy, the terms involving w are deleted from the preceding equation, as well as the product $\dot\eta_3\dot\eta_5$. F_v is decomposed into two separate forces

$$F_{33} = \tfrac{1}{2}\rho SC_D\dot\eta_3^2 \quad (31)$$

$$F_{55} = \tfrac{1}{2}\rho SC_D s^2\dot\eta_5^2 \quad (32)$$

To derive the equation for B^F_{33}, denote by $\hat z$ the amplitude of heave. Now $B^F_{33}\dot\eta_3$ represents a force, and during one heave cycle the following amount of work is done against this force

$$4\int_0^{\hat z} B^F_{33}\dot\eta_3 d\eta_3 = \pi\omega B^F_{33}\hat z^2 \quad (33)$$

This is equated to the work performed by the viscous force (31) during one cycle

$$4\int_0^{\hat z} F_{33}d\eta_3 = \frac{4}{3}\rho SC_D\omega^2\hat z^3 \quad (34)$$

This yields the following result for B^F_{33}:

$$B^F_{33} = (4/3\pi)\rho\omega\hat z C_D\Sigma S \quad (35)$$

In analogous fashion, an equation is derived for B^F_{55}

$$B^F_{55} = (4/3\pi)\rho\omega\hat\theta C_D\Sigma S|s|^3 \quad (36)$$

where $\hat\theta$ is pitch amplitude. $\hat z$ and $\hat\theta$ are, of course, unknown at the outset, and so their values must be estimated. Since foil damping at zero forward speed affects pitch and heave only slightly, more than one iteration is generally unnecessary.

Lateral Motions

The coupled sway, roll, and yaw equations are

$$(A_{22}+m)\ddot\eta_2 + B_{22}\dot\eta_2 + A_{24}\ddot\eta_4 + B_{24}\dot\eta_4 + C_{24}\eta_4$$
$$+ A_{26}\ddot\eta_6 + B_{26}\dot\eta_6 + C_{26}\eta_6 = F_2 e^{i\omega t} \quad (37)$$

$$A_{42}\ddot\eta_2 + B_{42}\dot\eta_2 + (A_{44}+I_4)\ddot\eta_4 + B_{44}\dot\eta_4 + C_{44}\eta_4$$
$$+ A_{46}\ddot\eta_6 + B_{46}\dot\eta_6 + C_{46}\eta_6 = F_4 e^{i\omega t} \quad (38)$$

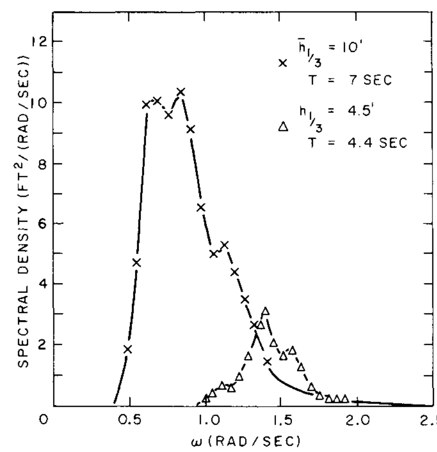

Fig. 3 Representative wave spectra *PHM* model tests.

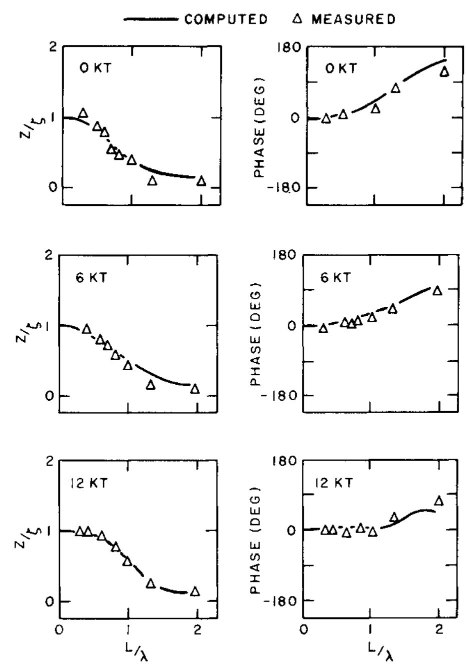

Fig. 4 *AG(EH)* heave in head seas.

$$A_{62}\ddot{\eta}_2+B_{62}\dot{\eta}_2+A_{64}\ddot{\eta}_4+B_{64}\dot{\eta}_4+C_{64}\eta_4+(A_{66}+I_6)\ddot{\eta}_6$$

$$+B_{66}\dot{\eta}_6+C_{66}\eta_6=F_6e^{i\omega t} \quad (39)$$

where I_4 and I_6 are the rolling and yawing moments of inertia, respectively.

Hull Coefficients

Again strip theory is used to evaluate the hull coefficients, and the reader is referred to Salvesen et al.[1] for derivation of the relevant mathematical expressions.

Foil coefficients

The starting point for deriving the lateral foil coefficients is to consider a foil of dihedral angle Γ and resolve its lift force L and moment N into sway, roll, and yaw components:

$$\text{sway force} = -L\sin\Gamma \quad (40)$$

$$\text{roll moment} = L\hat{y} \quad (41)$$

$$\text{yaw moment} = N\sin\Gamma \quad (42)$$

where

$$\hat{y}=y\cos\Gamma+z\sin\Gamma \quad (43)$$

y and z are coordinates of the foil's spanwise center of lift. Foil depth h also is referenced to this point. Here no distinction is made between foils and struts. The following sign convention is adopted for dihedral and anhedral angles: 1) for a port dihedral foil of angle Γ_{DP}, $\Gamma_i=\Gamma_{DP}$; 2) for a starboard dihedral foil of angle Γ_{DS}, $\Gamma_i=-\Gamma_{DS}$; 3) for a port anhedral foil of angle Γ_{AP}, $\Gamma_i=-\Gamma_{AP}$; and 4) for a starboard anhedral foil of angle Γ_{AS}, $\Gamma_i=\Gamma_{AS}$.

Denote by L_d and N_d the lift and moment acting on a foil as a result of swaying, yawing and rolling motions. In Eqs. (7) and (8), substitute $-\dot{\eta}_2\sin\Gamma+\hat{y}\dot{\eta}_4$ for $\dot{\eta}_3$, and $\eta_6\sin\Gamma$ for η_5, and hence obtain

$$L_{NC}=\pi\rho b(c/2)^2[(s\ddot{\eta}_6-U\dot{\eta}_6)\sin\Gamma+\ddot{\eta}_2\sin\Gamma-\hat{y}\ddot{\eta}_4] \quad (44)$$

$$L_C=½\rho USC_{L\alpha}C(k)$$
$$[\{[s-(c/4)]\dot{\eta}_6-U\eta_6\}\sin\Gamma+\dot{\eta}_2\sin\Gamma-\hat{y}\dot{\eta}_4]$$
$$-(\partial L/\partial h)C(k)y\eta_4 \quad (45)$$

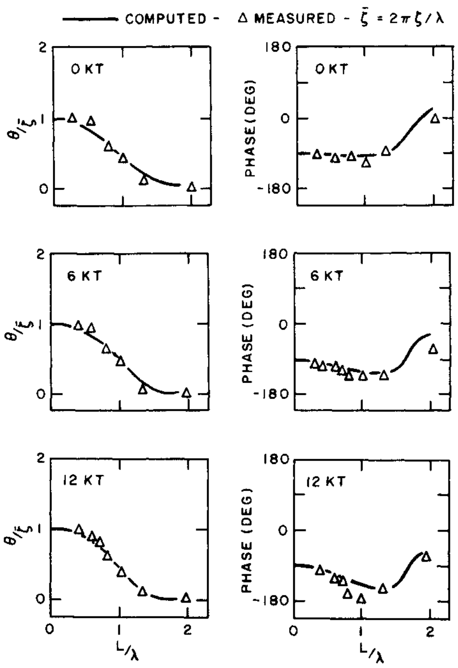

Fig. 5 ***AG(EH)* pitch in head seas.**

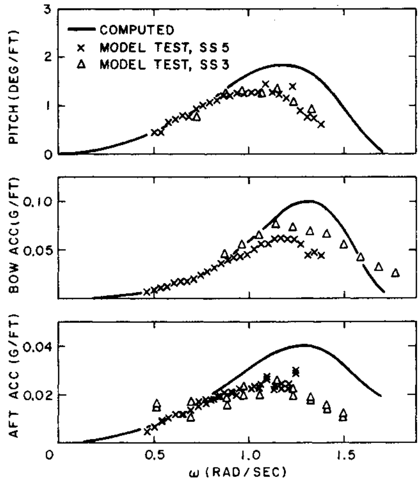

Fig. 6 Head sea response, 0 knots.

Similarly, from Eq. (6),

$$N_d=-L_{NC}s-L_Cx-\frac{\pi\rho bc^3\sin\Gamma}{16}(U\dot{\eta}_6+\frac{c}{8}\ddot{\eta}_6) \quad (46)$$

Consider now the foil exciting force and moment, and denote by L_w and N_w the lift and moment due to wave action on the foil. Then, from Eqs. (22) and (23)

$$L_w=½\rho USC_{L\alpha}G(k)\hat{w}+(\partial L/\partial h)C(k)\eta \quad (47)$$

$$N_w=-xL_w \quad (48)$$

where η is wave elevation and $\hat{w}$ the component of wave orbital velocity acting perpendicular to the foil

$$\hat{w}=w\cos\Gamma+u\sin\Gamma \quad (49)$$

where w is the vertical component and u the horizontal component. u is regarded as positive in the direction of propagation of the seaway. w is given by Eq. (24) and

$$u=\omega_w\exp(-k_wh)\eta \quad (50)$$

Substitution of Eqs. (44-50) into (40-42) yields the foil coefficients listed below. Summation is over all foil and strut elements:

$$A_{22}^F=\pi\rho\,\Sigma\,b(c/2)^2\sin^2\Gamma \quad (51)$$

$$B_{22}^F=½\rho U\,\Sigma\,SC_{L\alpha}C(k)\sin^2\Gamma \quad (52)$$

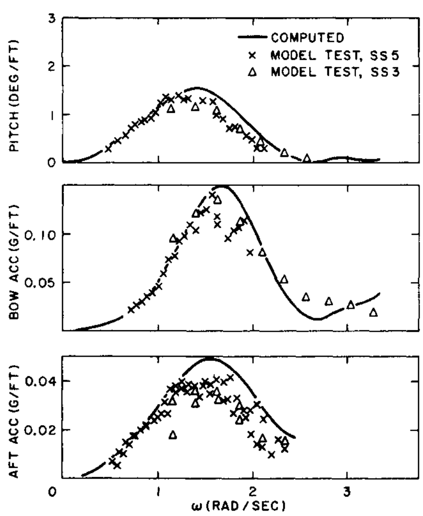

Fig. 7 Head sea response, 6 knots.

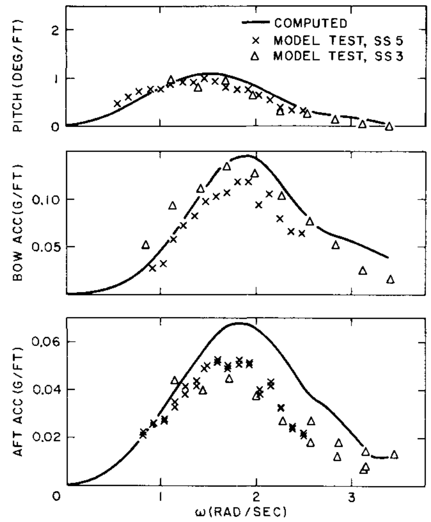

Fig. 8 Head sea response, 12 knots.

$$A^F_{24}=A^F_{42}=-\pi\rho\ \Sigma\ b(c/2)^2\hat{y}\sin\Gamma \tag{53}$$

$$B^F_{24}=B^F_{42}=-\tfrac{1}{2}\rho U\ \Sigma\ SC_{L\alpha}C(k)\hat{y}\sin\Gamma \tag{54}$$

$$C^F_{24}=-\tfrac{1}{2}\rho U^2\ \Sigma\ S(\partial C_L/\partial h)C(k)y\ \sin\Gamma \tag{55}$$

$$A^F_{26}=A^F_{62}=\pi\rho\ \Sigma\ b(c/2)^2 s\sin^2\Gamma \tag{56}$$

$$B^F_{26}=\rho U\ \Sigma\sin^2\Gamma\{-\pi b(c/2)^2+\tfrac{1}{2}SC_{L\alpha}C(k)[s-(c/4)]\} \tag{57}$$

$$C^F_{26}=-\tfrac{1}{2}\rho U^2\ \Sigma\ SC_{L\alpha}C(k)\sin^2\Gamma \tag{58}$$

$$A^F_{44}=\pi\rho\ \Sigma\ b(c/2)^2\hat{y}^2 \tag{59}$$

$$B^F_{44}=\tfrac{1}{2}\rho U\ \Sigma\ SC_{L\alpha}C(k)\hat{y}^2 \tag{60}$$

$$C^F_{44}=\tfrac{1}{2}\rho U^2\ \Sigma\ S(\partial C_L/\partial h)C(k)y\hat{y} \tag{61}$$

$$A^F_{46}=A^F_{64}=-\pi\rho\ \Sigma\ b(c/2)^2 s\hat{y}\ \sin\Gamma \tag{62}$$

$$B^F_{46}=\rho U\ \Sigma\hat{y}\sin\Gamma$$
$$\{\pi b(c/2)^2-\tfrac{1}{2}SC_{L\alpha}C(k)[s-(c/4)]\} \tag{63}$$

$$C^F_{46}=\tfrac{1}{2}\rho U^2\ \Sigma\ SC_{L\alpha}C(k)\hat{y}\sin\Gamma \tag{64}$$

$$B^F_{62}=\tfrac{1}{2}\rho U\ \Sigma\ xSC_{L\alpha}C(k)\sin^2\Gamma \tag{65}$$

$$B^F_{64}=-\tfrac{1}{2}\rho U\ \Sigma\ xSC_{L\alpha}C(k)\hat{y}\sin\Gamma \tag{66}$$

$$C^F_{64}=-\tfrac{1}{2}\rho U^2\ \Sigma\ xS(\partial C_L/\partial h)C(k)y\sin\Gamma \tag{67}$$

$$A^F_{66}=\pi\rho\ \Sigma\ [s^2 b(c/2)^2+(bc^4/128)]\sin^2\Gamma \tag{68}$$

$$B^F_{66}=\rho U\ \Sigma\ [-\pi b(c/2)^2+\tfrac{1}{2}xSC_{L\alpha}C(k)]$$
$$\times[s-(c/4)]\sin^2\Gamma \tag{69}$$

$$C^F_{66}=-\tfrac{1}{2}\rho U^2\ \Sigma\ xSC_{L\alpha}C(k)\sin^2\Gamma \tag{70}$$

$$F^F_2=\Sigma\ f^F_2\sin\Gamma \tag{71}$$

$$F^F_4=-\Sigma\ f^F_2\hat{y} \tag{72}$$

$$F^F_6=\Sigma\ f^F_2 x\sin\Gamma \tag{73}$$

where f^F_2 is the sway exciting force acting on an individual foil element

$$f^F_2=-\tfrac{1}{2}\rho USC_{L\alpha}G(k)\omega_w(\sin\Gamma\ \sin\beta_s+i\ \cos\Gamma)$$
$$\exp\{-k_w[h+i(x\ \cos\beta_s-y\ \sin\beta_s)]\} \tag{74}$$

The foil damping, restoring, and exciting force coefficients just given apply only when $U<0$. For $U=0$, the foil restoring and exciting force terms are neglected, whereas damping coefficients are calculated by considering the viscous drag forces opposing lateral motions which act on the foils. The procedure is similar to the derivation of Eq. (35) and yields the following result for the viscous roll damping coefficient

$$B^F_{44}=(4/3\pi)\rho\omega\hat{\eta}_4\ \Sigma\ (y^2+z^2)^{3/2}SC_n\sin\alpha \tag{75}$$

where $\hat{\eta}_4$ is roll amplitude and C_n is the normal force coefficient for a flat plate tilted at angle α to the flow. From Hoerner,[6]

$$C_n=\begin{cases}0.0467\ \alpha, & \alpha<40^\circ\\ 1.17, & \alpha>40^\circ\end{cases} \tag{76}$$

and, from geometrical considerations,

$$\tan\alpha=\left|\frac{y/z+\tan\Gamma}{1-(y/z)\tan\Gamma}\right| \tag{77}$$

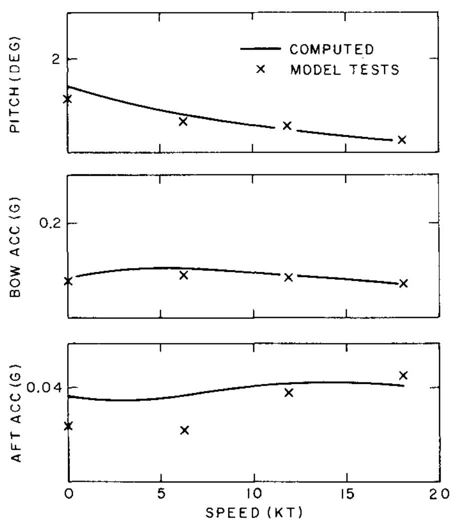

Fig. 9 Rms vertical motions in head sea state 3.

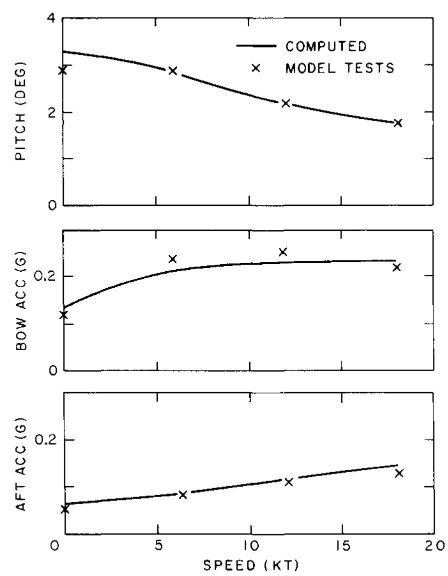

Fig. 10 Rms vertical motions in head sea state 5.

Table 1 *Bras d'Or* **rms vertical accelerations**

	Sea state 5 12.5 knots		Sea state 6 8.0 knots	
	Computed	Measured	Computed	Measured
Bow, g	0.12	0.11	0.19	0.18
Stern, g	0.086	0.074	0.13	0.14

Similar equations may be derived for other foil damping terms, but these are much less significant than the viscous roll damping term.

One additional source of damping remains to be considered. A strut in or near the free surface will generate waves when oscillated laterally. The resultant damping terms due to wave-making affect roll appreciably at low speeds. For a vertical strut, the sway wave-making damping term is[7]

$$B_{22}^{W} = (\pi/2)\rho\omega b^2 c\, C_w \tag{78}$$

where C_w is a function of $\omega^2 b/g$. Roll and yaw wave-making damping terms are obtained by multiplying B_{22}^{W} by the appropriate foil coordinates.

III, Comparison of Theory with Experiment

The theoretical predictions will be compared with experimental data for three very different hydrofoil craft. Two are the U.S. hydrofoils *AG(EH)* and *PHM*, for which model tests are the source of data. Both have fully submerged foil systems that differ considerably in detail (one is airplane, the other canard); their hulls are also considerably different. The third ship is the Canadian hydrofoil *Bras d'Or*, with a slender hull and a surface-piercing foil system, for which open-water full-scale trials data are available.

In the case of *AG(EH)*, the model tests were carried out at the U.S. Naval Ship Research and Development Center using a self-propelled model in regular waves.[9] The *PHM* model tests were conducted at the Davidson Laboratory, using a towed model in irregular waves[10]; representative wave spectra, as measured during the tests and converted to full scale, are shown in Fig. 3.

Heave and Pitch Comparisons

Heave and pitch response data for *AG(EH)* are compared with theoretical predictions in Figs. 4 and 5. Data are given for three speeds (0, 6, and 12 knots full scale) in head seas. Heave is nondimensionalized by wave amplitude and pitch by wave slope. Agreement between theory and experiment is very good, both for amplitudes and phases.

For *PHM*, head sea pitch angle and vertical acceleration frequency response data are given by Henry[10] for full-scale speeds of 0, 6, and 12 knots. These measurements are compared with predictions in Figs. 6-8. In each case, the triangles and crosses represent data obtained in sea states 3 and 5, respectively. It is worth noting that the response operators for the two sea states are in reasonable agreement; in fact, Henry states that this agreement indicates that wave-induced motions vary linearly with wave height up to sea state 5. This supports the assumption of linearity made in developing the prediction theory.

Predicted and measured frequency responses for *PHM* are in good agreement at 6 and 12 knots, particularly for pitch and bow acceleration; but, at zero speed, predicted response is excessive at frequencies greater than about 1.0 rad/sec. This discrepancy is particularly evident for aft acceleration. The cause may be underestimation of foil viscous damping; however, the fact that both heave and pitch are very well predicted for *AG(EH)* at zero speed (Figs. 4 and 5) makes interpretation of the discrepancies in Fig. 6 rather difficult.

Figures 9 and 10 compare predicted and measured rms vertical motions for *PHM* in sea states 3 and 5, respectively. The wave spectra of Fig. 3 have been used in making the predictions. Agreement is generally very good, with the single exception that predictions of aft acceleration are excessively high at low speed in sea state 3. In sea state 5, on the other hand, aft acceleration is very well predicted across the speed range. Since the sea state 5 spectrum of Fig. 3 is reasonably representative of full-scale environmental conditions of operational interest, whereas the sea state 3 spectrum is not, Fig. 10 suggests that *PHM* response is well predicted over the range of significant seaway energy.

The full-scale trials of *HMCS Bras d'Or* are the other source of head sea vertical motion data. Table 1 compares the

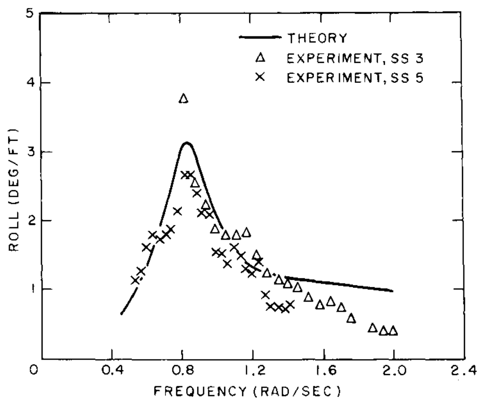

Fig. 11 Beam sea roll response, 0 knots.

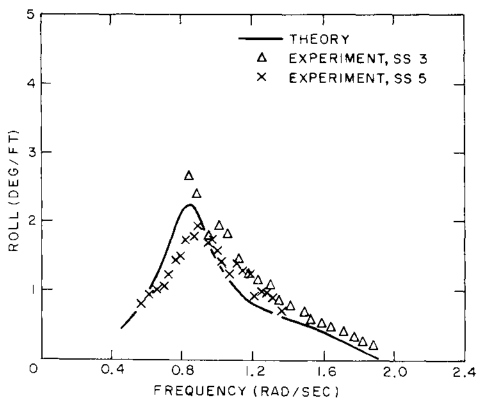

Fig. 12 Bow sea roll response, 0 knots.

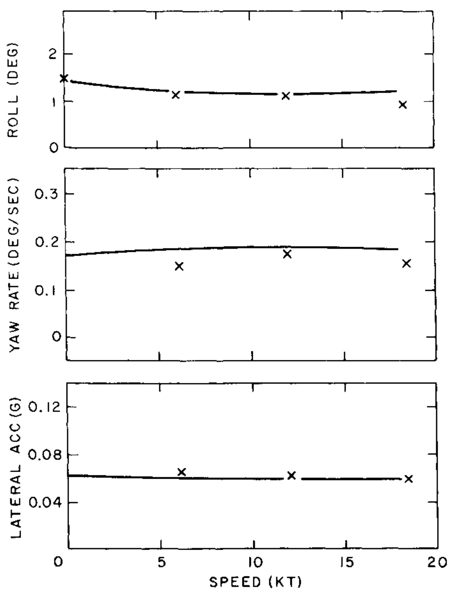

Fig. 13 Rms lateral motions in beam sea state 3.

limited available trials data with theoretical estimates. Again, agreement is good.

Sway, Roll, and Yaw Comparisons

Lateral motion data from the *PHM* model tests are compared with predictions in Figs. 11-13. Unfortunately, because of towing tank test restrictions, the only useful frequency response measurements are for rolling at zero speed. Figures 11 and 12 show generally satisfactory agreement between predicted and measured roll response at zero speed in beam and bow seas. Beam sea rolling is over predicted in the frequency range 1.5-2.0 rad/sec; but, as seaways of practical interest have little energy in this band, this is not a serious shortcoming. One reasonably may conclude from these comparisons that hove-to rolling predictions should be satisfactory.

Predicted and measured rms lateral motions in beam sea state 3 are compared in Fig. 13. Agreement is satisfactory, but the somewhat academic nature of this spectrum (Fig. 3) does not permit generalizations based on these results, since little seaway energy is present in the frequency range of greatest interest (0.3 to 1.5 rad/sec).

IV. Conclusions

A mathematical model has been developed for prediction of wave-induced motions for hullborne hydrofoils. The generally good agreement just demonstrated between predicted and measured vertical motions suggest that this model's heave and pitch predictions for hullborne hydrofoils should be of comparable accuracy to strip theory predictions for conventional displacement hulls. As to lateral motions, although predictions agree well with the measurements given, the experimental data are not sufficiently extensive to permit meaningful assessment of the general reliability of the predictions.

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